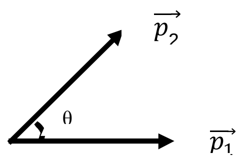
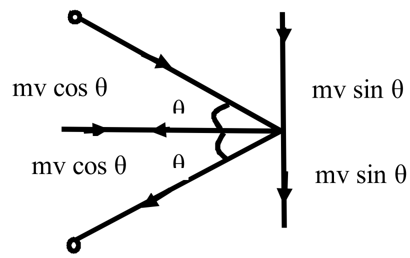
LAWS OF MOTION
FRICTION

NEWTON'S LAWS OF MOTION

- Inertia: The inability of a body to change by itself its state of rest uniform motion.
- Mass is a measure of inertia.
- Momentum : The ability of a body to produce motion. It is the product of mass and velocity. **P = mv**. It is a vector quantity. Unit = Kg-m/s



$\vec{p} = \vec{p_1} + \vec{p_2}$



Ball hitting a wall at an angle θ

- Resultant momentum of two bodies of moments p_1 and p_2 is $\sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta}$
- Change in momentum $\Delta\vec{p} = \vec{p_2} - \vec{p_1}$, where p_2 and p_1 are the final and initial moments.
- When a ball hits a wall with a velocity v making an angle θ and rebounds with the same velocity making the same angle with the wall, then the change in momentum of the ball is equal to 2mv cos θ in the normally outward direction from the wall.
- When two bodies of momenta **p** each move perpendicular to each other the resultant momentum is √2 **p.**
- When two bodies of momenta **p** each move with an angle of 120º to each other the resultant momentum is p**.**

  Newton's first law of motion: Every Body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

  If the net external force on a body is zero, its acceleration is zero. If there is net external force the body has non zero acceleration.
- Newton's second law: The rate of change of momentum is directly proportional to external force and takes place in the direction of force.
- $F_R = m\frac{\Delta v}{\Delta t} + v\frac{\Delta m}{\Delta t}$
- If m is constant $F_R = m\frac{\Delta v}{\Delta t} = ma$
- If v is constant $F_R = v\frac{\Delta m}{\Delta t}$
- If F_R is equal to zero, the body will be either at rest or moves with constant velocity. This is the first law.
- If F_R is equal to zero, the forces on the body will be equal and opposite. This is the third law.

1. If $\vec{F}$ is equal to zero, the body will be either at rest or moves with constant velocity. This is the first law. If $\vec{F}$ is equal to zero the force on the body will be equal and opposite. This is the third law.
2. The second law of motion is a vector law. It is equivalent to three equations.

$$\sum F_x = \frac{dp_x}{dt} = ma_x \qquad \sum F_y = \frac{dp_y}{dt} = ma_y \qquad \sum F_z = \frac{dp_z}{dt} = ma_z$$

3. The second law of motion is applicable to a single particle. The force $\vec{F}$ in the law stands for the net external force on the particle and $\vec{a}$ stands for the acceleration of the particle. Any internal forces in the system are not to be included in $\vec{F}$.

- Force: There are four fundamental forces in nature. These are strong, electromagnetic, weak and gravitational forces. Their relative magnitudes are in the ratio $1:10^{-2}:10^{-13}: 10^{-38}$.
- The weakest force in nature is gravitational force.
- If the force is applied in the direction of motion then there is change in the magnitude of velocity and the body undergoes linear motion.
- If the force is applied perpendicular to the motion of the body, then there is change in the direction of velocity only and the body undergoes uniform circular motion.
- If the force acts at an angle to the motion, then there is change in the magnitude and direction of velocity and the body undergoes non-uniform circular motion.
- Resultant of two forces $F = \sqrt{F_1{}^2 + F_2{}^2 + 2F_1F_2\cos\theta}$
  - Newton's third law of motion: To every action, there is always an equal and opposite reaction.

Points:

  - Forces always occur in pairs. Force on a body A by B is equal and opposite to force on the body B by A.
  - The force on A by B and the force on B by A act at the same instant.
  - Action and reaction forces act on different bodies, not on the same body. Consider a pair of bodies A and B. Then according to Newton's third law $\vec{F}_{AB} = -\vec{F}_{BA}$
  - Force on A by B=-force on B by A

Common forces in mechanics

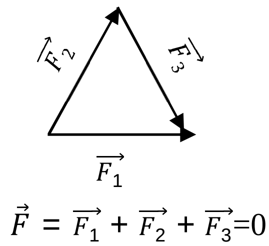
- Weight:
  The force with which the earth attracts a body is called gravitational force.
- It is a vector quantity.
- In vector form gravitational force is given by $\vec{F_g} = m\vec{g}$
- Weight W of a body is the magnitude of the net force required to prevent a body from falling freely, as measured by someone on the ground.
- The weight of a body is equal to the magnitude of F_g of the gravitational force on the body.
- $\vec{W} = m\vec{g}$ or $-W\hat{j} = -mg\hat{j}$
- W = mg
- Unit = Newton. Practical unit = Kg. wt

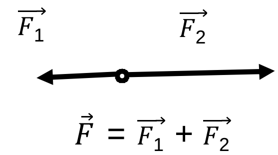
- Apparent weight: The weight of the body must be measured when the body is not accelerating vertically relative to the ground. If the measurement of weight of a body is done in an accelerating elevator, the reading differs from the weight that we measure without acceleration. Such a measurement is called apparent weight.
- NOTE: Weight changes with g but mass is constant. It is proportional to the mass. Mass is constant everywhere whereas weight changes
- Contact force: When two surfaces are in contact they exert forces on each other. Such forces are called contact forces. The contact forces are resolved into components one parallel to the surface and the other perpendicular to the surface. The force perpendicular to the surface is called normal reaction. The force parallel to the surface is called friction. Tension is also a contact force. Spring force is a contact force.
- Normal force: if we stand on a mattress, Earth pulls us downwards, but we are stationary because the mattress deforms downward due to us, pushing us up. Even a concrete floor pushes us up because it also deforms due us standing on it and pushes us up. This push on us from the floor or mattress is called normal force. It is represented by $\vec{N}$. It always acts perpendicular to the surface.
- Friction: A frictional force is a force that opposes the attempted slide of a body over a surface. It is represented by $\vec{f}$.
- Tension:
- Any long thread rope or wire can be called as a string. When a string attached to a body is taut, the string pulls on the body with a force $\vec{T}$ directed away from the body along the string. This force is called a tension force or simply tension.
- If string is assumed to be mass less then tension in the string is uniform everywhere. If the string has mass tension at different points will be different. Maximum tension that a string can bear is called breaking strength.
- When a rope of mass M has length L, then the mass of a part of rope of length is $\frac{M}{L}l$. Here M/L is the mass per unit length.
- Spring force: A spring is coiled wire which is stretchable or compressible. Springs are assumed to be massless. Elastic force in a spring is assumed to be same everywhere in the spring. When a spring is stretched or compressed, the elongation or compression in the spring is directly proportional to the force acting on the spring according to Hooke's law. Spring force is given by $\overrightarrow{F_s} = -k\vec{x}$ (Hooke's law). Here k is a constant called spring constant or force constant. It is a measure of stiffness of the spring. Its unit is N/m. D.F= MT^{-2}.
- If x is positive, then spring force is negative and vice versa.
- Spring force is inversely proportional the length of the spring.
- Law of conservation of momentum: If there is no external force acting on the body, the total vector momentum of an isolated system remains constant.
- If $\overrightarrow{F_R}$= 0, $\frac{d\vec{P}}{dt}$ = 0 and $\vec{P}$= constant.

Equilibrium of a particle

- Equilibrium of a particle refers to the situation when the net external force on the particle is zero.
- If two forces $\vec{F}_1$ and $\vec{F}_2$, act on a particle, equilibrium requires

$$\vec{F}_1 = -\vec{F}_2$$





Equilibrium of a particle

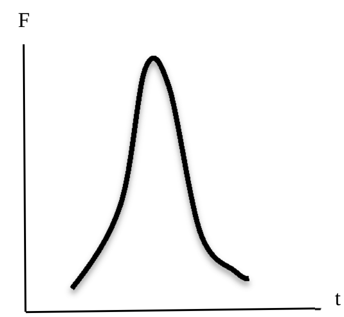
- Equilibrium under three concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ required that the vector sum of the three forces is zero.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

- The above equation can be written in component form as follows

$$F_{1x} + F_{2x} + F_{3x} = 0 \qquad F_{1y} + F_{2y} + F_{3y} = 0 \qquad F_{1z} + F_{2z} + F_{3z} = 0$$

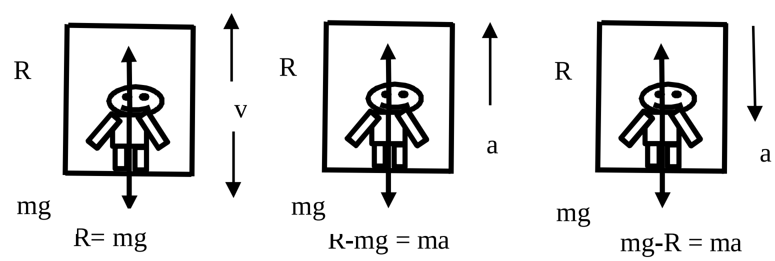
- Impulse:
  The product of force and time is called impulse.
- I = F t.
- unit = N-s. It is a vector quantity.
- Impulse = change in momentum = mv - mu
- Impulsive force: A large force acting for a small time.



F-t graph

- Area under the force-time graph gives magnitude of impulse.

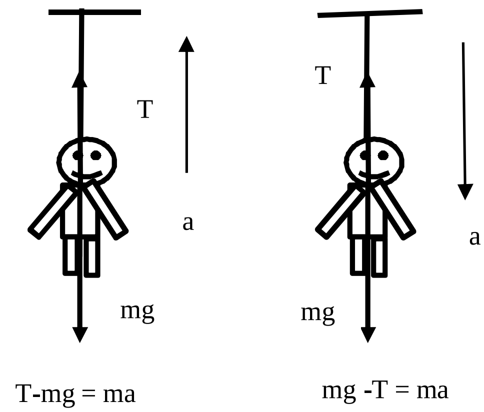
Apparent weight of a person in a lift:



Man in a lift

- When a lift is at rest or moving upwards or downwards with constant velocity, then the apparent weight of a person of mass m is R = mg.
- When a lift is moving upwards with constant acceleration, then the apparent weight of a person of mass m is R = mg + ma.
- When a lift is moving downwards with constant acceleration, then the apparent weight of a person of mass m is R = mg-ma.
- When the lift is falling freely, apparent weight R = 0.

- If a person of mass m is hanging from a rope, the tension in the rope is equal to the weight of the person. T = mg
- If the person is sliding down the rope with an acceleration a, then the tension in the rope is T = mg - ma
- If the person is moving up the rope with an acceleration a, then the tension in the rope is T = mg +ma
- If the person falls freely then T = 0.



Man on a rope

- If a gun fires n bullets each of mass m per second with velocity u, the force required to hold the gun in position is F = mnu.

- If a hose pipe of area of cross section A delivers water of density with velocity v, the force required to hold the pipe is F = $Av^2\rho$.
- When water coming out of a hose pipe of area of cross section A with a speed v and hits a wall normally and rebounds with the same speed, the force exerted by the water on the wall is $2Av^2\rho$.
- When water coming out of a hose pipe of area of cross section A with a speed v, hits a wall at an angle θ to the normal to the wall and rebounds with the same speed making the same angle, the force exerted by the water on the wall is $2Av^2\rho \cos\theta$.
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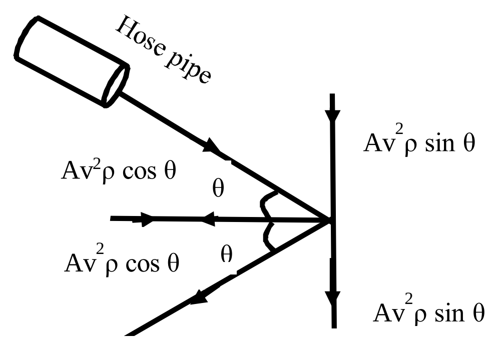
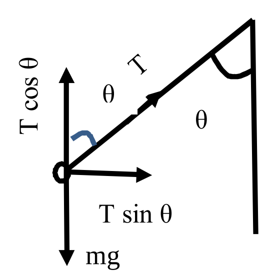


Water hitting a wall at an angle θ and rebounding with same speed

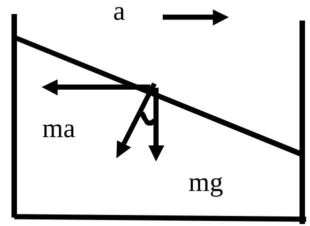
FRAMES OF REFERENCE:

- <u>Inertial frame of reference</u>: A frame in which the Newton's laws of motion are valid. Ex. A body at rest or moving with constant velocity.
- <u>Non-inertial frame of reference</u>: A frame in which the Newton's laws of motion are not valid. Ex. Any accelerating body like earth.
- <u>Pseudo forces</u>:
- To apply Newton's laws of motion in non-inertial frame a pseudo force is imagined opposite to the direction of motion. Ex. Centrifugal force.

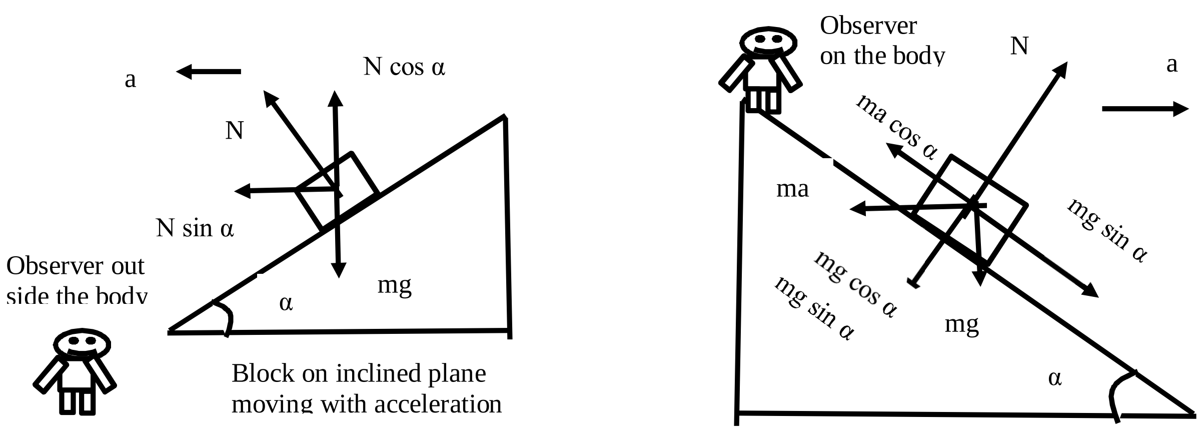


- A simple pendulum is suspended in a carriage moving with acceleration a in the horizontal direction. Then the tension in the string is given by $T = \sqrt{g^2 + a^2}$ and the angle made by the pendulum with the vertical is tan θ =a/g.

- A vehicle carries a tank of water. If the vehicle moves with a uniform acceleration a in the horizontal direction the angle made by the surface of water in the tank with horizontal is $\tan\theta = \frac{a}{g}$.



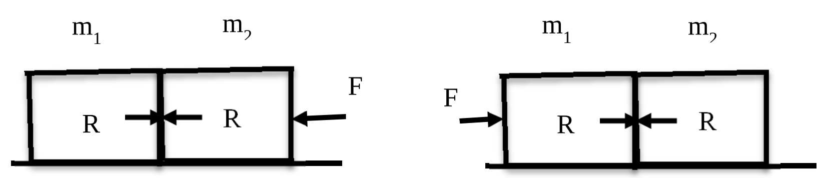
<u>Acceleration of an inclined plane to keep a block on it stationary</u>



Block on inclined plane moving with acceleration

Block on inclined plane moving with acceleration

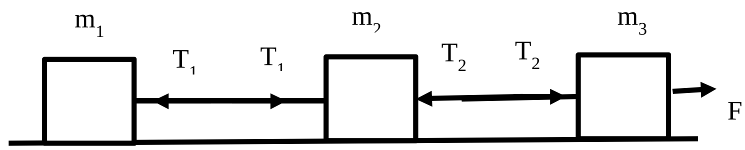
- A block is kept on a frictionless inclined surface with angle of inclination α. **The incline is** given an acceleration 'a' to keep the block stationary. Then a = g tanα.
- <u>Motion of connected bodies</u>
- <u>Two blocks in contact with each other</u>
- Consider two blocks of masses m_1 and m_2 put in contact with each other and a horizontal force is applied on the block of mass m_1. Then



Two blocks in contact

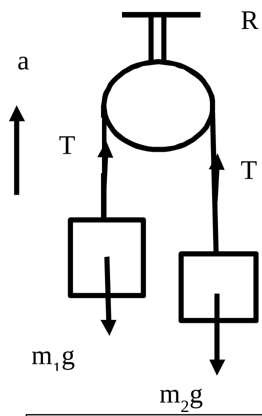
- $F - R = m_1 a$ and $R = m_2 a$ $F = (m_1 + m_2)a$, $a = \frac{F}{m_1+m_2}$,
- $R = \frac{m_2 F}{m_1+m_2}$
- Consider two blocks of masses m_1 and m_2 put in contact with each other and a horizontal force is applied on the block of mass m_2. Then
- $F - R = m_2 a$ and $R = m_1 a$ $F = (m_1 + m_2)a$, $a = \frac{F}{m_1+m_2}$, $R = \frac{m_1 F}{m_1+m_2}$
- <u>Three blocks connected by strings</u>

- Consider three masses m_1,m_2 and m_3 connected by strings and string of mass m_3 is pulled by a horizontal force F. Then



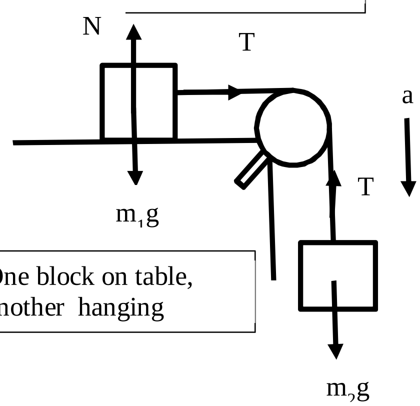
Three blocks connected by strings

- $a = \frac{F}{m_1+m_2+m_3}$,$T_1 = m_1 a = \frac{m_1 F}{m_1+m_2+m_3}$,$T_2 - T_1 = m_2 a$, $F - T_2 = m_3 a$, $T_1 = \frac{(m_1+m_2)F}{m_1+m_2+m_3}$
- <u>Atwood's machine</u>
- Consider two bodies of masses m_1 and m_2 connected by a light string passing over a massless pulley. If $m_2 > m_1$ m_2 moves down and m_1 moves up. Then



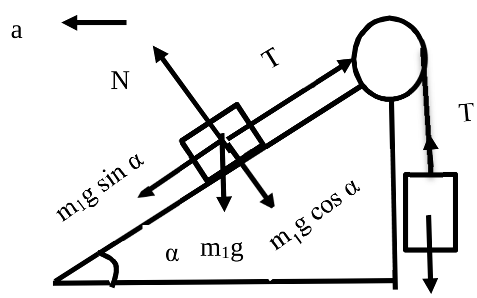
Atwood's machine

- $m_2 g - T = m_2 g$,$T - m_1 g = m_1 g$,$a = \frac{(m_2-m_1)g}{m_1+m_2}$,$T = \frac{2m_1 m_2 g}{m_1+m_2}$
- Reaction at the pulley $R = 2T = \frac{4m_1 m_2 g}{m_1+m_2}$
- <u>Two blocks connected and one block placed on horizontal plan</u>
- Consider two bodies of masses m_1 and m_2 connected by a light string passing over a massless pulley and one block is placed on inclined plane of inclination α . If $m_2 > m_1$ m_2 moves down and m_1 moves up. Then



One block on table, another hanging

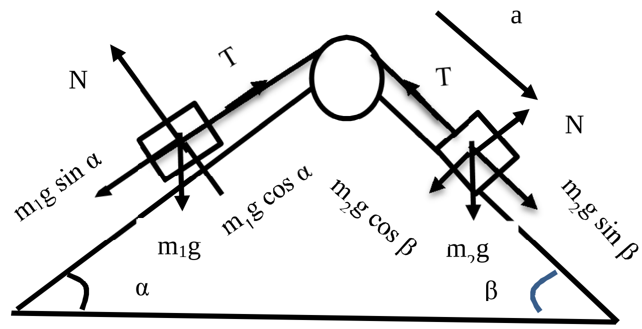
- $m_2 g - T = m_2 a$,$T = m_1 a$,$a = \frac{m_2 g}{m_1+m_2}$,$T = \frac{2m_1 m_2 g}{m_1+m_2}$
- Reaction at the pulley $R = \sqrt{2}T$
- <u>Two blocks connected and one block placed on inclined plane</u>
  - Consider two bodies of masses m_1 and m_2 connected by a light string passing over a massless pulley and one block is placed on inclined plane of inclination α . If $m_2 > m_1$ m_2 moves down and m_1 moves up. Then



One Block on inclined plane and another hanging

- $m_2 g - T = m_2 a$,$T - m_1 g \sin\alpha = m_1 a$,$a = \frac{(m_2 - m_1 \sin\alpha)g}{m_1+m_2}$,$T = \frac{2m_1 m_2 (1+\sin\alpha)g}{m_1+m_2}$

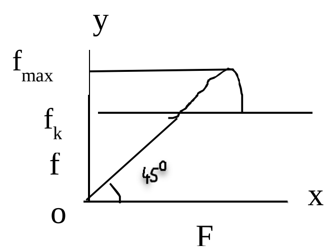
- Reaction at the pulley $R = 2T \cos\frac{90-\alpha}{2}$
- <u>Two blocks connected and placed on inclined planes</u>
- Consider two bodies of masses m_1 and m_2 connected by a light string passing over a massless pulley and placed on inclined planes of inclinations α and β. If $m_2 > m_1$ m_2 moves down and m_1 moves up. Then
- $m_2 g \sin\beta - T = m_2 a$,
- $T - m_1 g \sin\alpha = m_1 a$,
- $a = \frac{(m_2 \sin\beta - m_1 \sin\alpha)g}{m_1 + m_2}$,
- $T = \frac{2m_1 m_2(\sin\alpha + \sin\beta)g}{m_1+m_2}$



Both blocks on inclined planes of inclination α and β

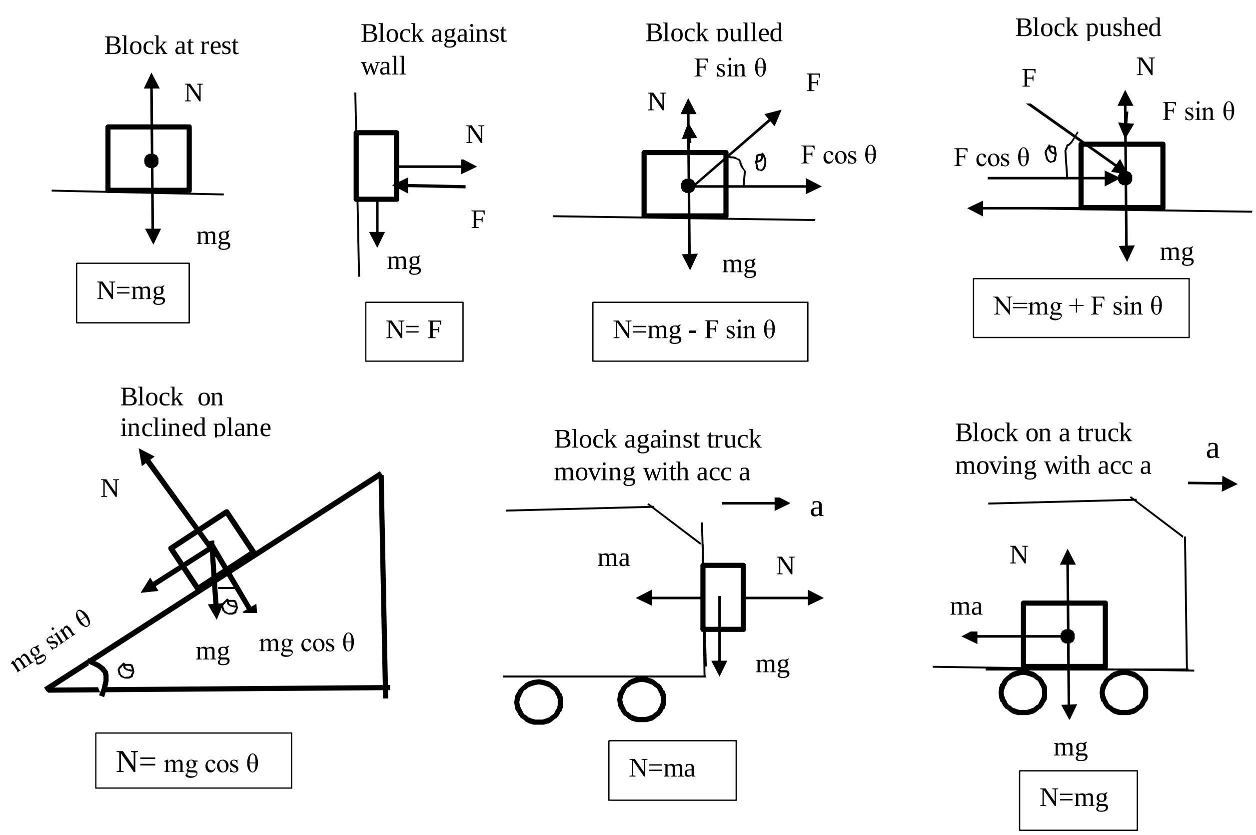
FRICTION

- <u>Friction</u>: The force that opposes the motion of one body over the surface of another body is called frictional force or simply friction.
- <u>Normal reaction</u>: The force that acts normal to the surface on which the body is placed.
- <u>Static friction</u>: The force that acts between two surfaces when the body is at rest and this acts only when the body is subjected to an external force.
- <u>Kinetic friction</u>: The force that acts between two surfaces when one body moves on the surface of the other body.
- <u>Rolling friction</u>: The force that acts between two surfaces when one body rolls on the surface of the other body.
- Frictional force is due to irregularities in the surfaces.
- <u>Static friction</u>: Static frictional force is a self-adjusting force. As the external force is increased, it increases and becomes maximum. This maximum force is called as maximum force of static friction or limiting friction.
- <u>Laws of static friction</u>:
- Friction always acts parallel to the surfaces in contact.
- It is independent of the area of contact.
- Limiting friction is directly proportional to the normal reaction
- $f_s = \mu_s R$
- Coefficient of static friction can be less than or greater than one. It has no units and dimensions.
- For copper and copper μ_s = is 1.60
- For glass and glass μ_s = 1.00
- For ice and ice μ_s = 0.1



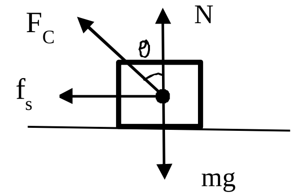
Variation of with external force F

- <u>Laws of kinetic friction</u>: It is independent of the relative speed between the surfaces.
- Kinetic friction is directly proportional to the normal reaction $f_k = \mu_k R$
- A graph drawn between applied force on the body and the frictional force is as shown in figure.
- Rolling friction is directly proportional to the normal reaction $f_R = \mu_R R$
- Small break or large break, both are equally effective in stopping a bicycle since μ is independent of area of contact.
- When a man is walking forward frictional force also acts forwards.
- When a person is pedaling a bicycle and the bicycle is moving forwards, the frictional force will act on the front wheel in the backward direction and on the back wheel in the forward direction.
- $\mu_S > \mu_K > \mu_R$
- <u>Calculation of normal reaction</u>:



- When a body of mass m is placed on a level surface N = mg
- When a body of mass m placed on a level surface is pulled by a force F making an angle θ with the horizontal N= mg – F sin θ
- When a body of mass m placed on a level surface is pushed by a force F making an angle θ with the horizontal N= mg + F sin θ
- When a body of mass m is placed on an incline of inclination θ N = mg cos θ
- When a body is pushed against a wall with a force F N = F

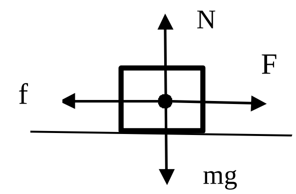
- <u>Angle of friction</u>: Angle that the resultant of normal reaction and the frictional force makes with the normal reaction.
- When a body of mass m is subjected to a horizontal force F, it tends to slide on a level surface.



Then $\mu_s = \tan\theta = \frac{f_s}{R} = \frac{F}{mg}$ where θ is the angle of friction.

<u>Motion of a body on a rough level surface</u>:

static frictional force is $f_s = \mu_s mg$.
If external force $F < f_s$, then the frictional force is $f = F$. The body does not slide.
If external force $F = f_s$, then the frictional force is $f = f_s = F = \mu_s mg$. The body tends to slide.
If external force $F > f_s$, then the frictional force is $f = f_k = \mu_k mg$. The body slides with acceleration.

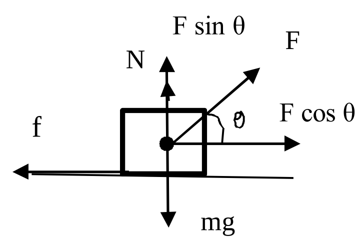


Motion on rough level surface

When a body is subjected to a horizontal force F on a rough level surface, the acceleration acting on the body is $a = \frac{F - f_k}{m} = \frac{F - \mu_k mg}{m}$

<u>Pulling</u>

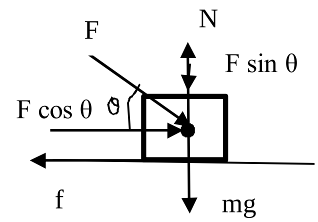
Block pulled



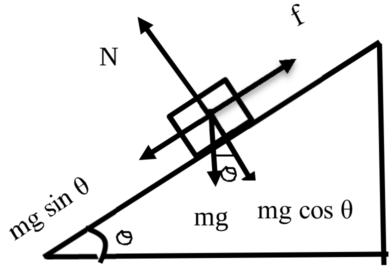
- When a block is pulled with a force F at an angle θ on a rough level surface, static frictional force on the block is $f_s = \mu_s(mg - F\sin\theta)$. External force is $F = F\cos\theta$.
- If the external force $F\cos\theta < f_s$, then f $= F\cos\theta$. The block does not slide.
- If the external force $F\cos\theta = f_s$, then $f_s = F\cos\theta = \mu_s(mg - F\sin\theta)$. The block tends to slide.
- If the external force $F\cos\theta > f_s$, then $f = f_k = \mu_k(mg - F\sin\theta)$. The block moves with acceleration. Then net force $F_R = F\,cos\,\theta - \mu_k(mg - F\,sin\,\theta)$. $if\ F_R =$ 0, $then\ F\cos\theta = \mu_k(mg - F\sin\theta)$
- <u>Pushing</u>
- When a block is pulled with a force F at an angle θ on a rough level surface, static frictional force on the block is $f_s = \mu_s(mg + F\sin\theta)$. External force is $F = F\cos\theta$.

- If the external force $F\cos\theta < f_s$, then f =$F\cos\theta$. The block does not slide.
- If the external force $F\cos\theta = f_s$, then $f_s = F\cos\theta = \mu_s(mg + F\sin\theta)$. The block tends to slide.
- If the external force $F\cos\theta > f_s$, then $f = f_k = \mu_k(mg + F\sin\theta)$. The block moves with acceleration. Then net force $F_R = F\,cos\,\theta - \mu_k(mg + F\,sin\,\theta)$. $if\ F_R = 0$, $then\ F\cos\theta = \mu_k(mg + F\sin\theta)$

Block pushed

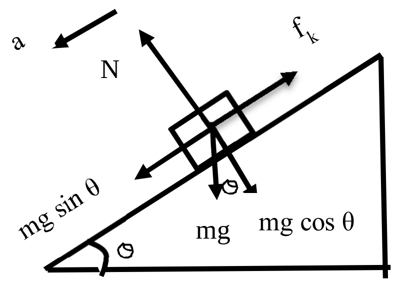


- Motion of a body on an inclined rough surface:



Block stationary on inclined plane

a=0

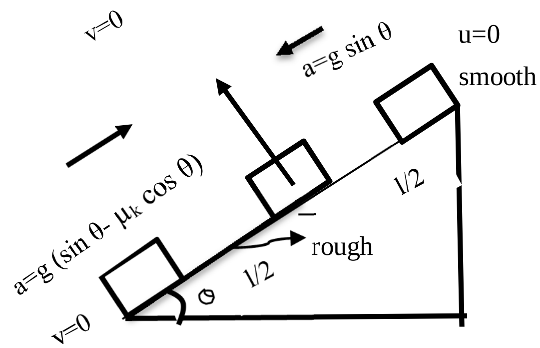


a=g (sin θ- μ_k cos θ)

Block sliding down on rough inclined plane

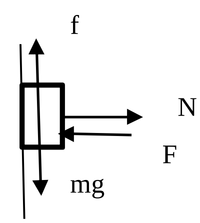
- Consider a body of mass m is placed on a rough inclined surface of inclination θ.
- Static frictional force $f_s = mg\sin\theta$.
- If the external force $mg\sin\theta < f_s$, then fricitonal force $f = F = mg\sin\theta$
- If the external force $mg\sin\theta = f_s$, then fricitonal force $f = f_s = F = mg\sin\theta$. The body tends to slide. The angle of inclination for which the body tends to slide is called angle of repose. Then $f_s = F = mg\sin\alpha = \mu_s mg\cos\alpha$ where α is called as angle of repose and θ = α
- If the external force $mg\sin\theta > f_s$, then the body slides with acceleration. Then the frctionaal force on the body is $f = f_k = \mu_k mg\cos\theta$. When a body moves down a rough inclined surface of inclination θ, $F_R = mg\sin\theta - f_k = mg\sin\theta - \mu_k mg\cos\theta$. $a = g(\sin\theta - \mu_k\cos\theta)$

- When a body is moving down a rough inclined surface of length l starting from rest, the time taken to move down the incline is $t = \sqrt{\frac{2l}{g(\sin\theta - \mu_k \cos\theta)}}$
- When the body moves down a smooth inclined plane, $a = g\sin\theta$
- When a body is moving down a smooth inclined surface of length l starting from rest, the time taken to move down the incline is $t = \sqrt{\frac{2l}{g\sin\theta}}$
- The upper half of an inclined plane with inclination θ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by μ = 2 tanθ.



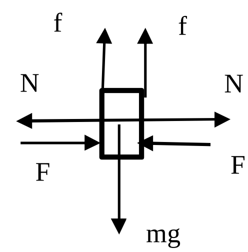
Upper half smooth and lower half rough

- A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is $\mu_K = 1 - \frac{1}{n^2}$
- <u>Duster against blackboard</u>
  - When a duster is pressed against a wall with a force F then
  - Static frictional force $f_s = \mu_s N = \mu_s F$
- External force = mg
- If external force $mg < f_s$, then the frictional force is $f = mg$. The duster does not slide.
- If external force $mg = f_s$, then the frictional force is $f = f_s = mg = \mu_s N = \mu_s F$. The duster tends to slide.
- If external force $mg > f_s$, then the frictional force is $f = f_k = \mu_k F$. The duster slides with acceleration.



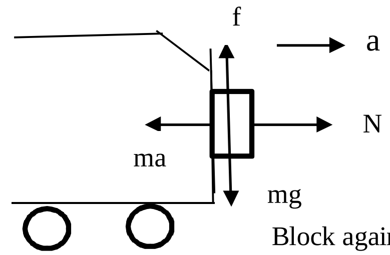
Duster against black board

- <u>Book held by both the hands</u>
- When a book is held by both the hands, the force applied by each hand is F.
- Static frictional force on hand $f_s = \mu_s N = \mu_s F$. Total upward static fricitonal force $2f_s = 2\mu_s N = 2\mu_s F$
- External force = mg
- If external force $mg < 2f_s$, then the frictional force is $f = mg$. The book does not slide.
- If external force $mg = 2f_s$, then the frictional force is $f = 2f_s = 2\mu_s N = 2\mu_s F = mg$. The book tends to slide.



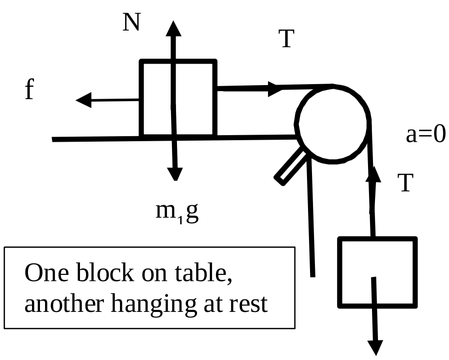
Book held by both hands

- If external force $mg > 2f_s$, then the frictional force is $f = 2f_k = 2\mu_k F$. The book slides with acceleration.
- <u>Block against a bus</u>
- A bus is moving on a level road with acceleration. A block of mass m is struck to the front part of the bus. The coefficient of friction between the bus and the block is μ.

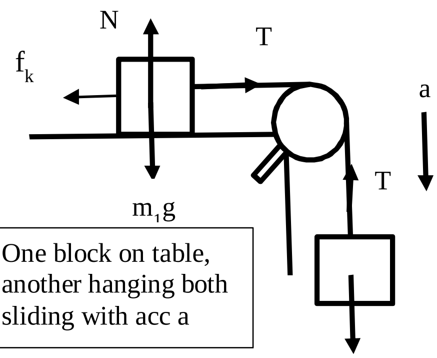


Block against truck moving with acc a

- Static frictional force is $f_s = \mu_s N$.
- External force = mg
- If external force $mg < f_s$, then the frictional force is $f = mg$. The block does not slide.
- If external force $mg = f_s$, then the frictional force is $f = f_s = mg = \mu_s N = \mu_s ma$. The block tends to slide.
- If external force $mg > f_s$, then the frictional force is $f = f_k = \mu_k ma$. The block slides with acceleration.
- The block tends to slide if $f_s = mg = \mu_s ma$
- The minimum acceleration with which the bus should move so that the block does not slide is $a = \frac{g}{\mu_s}$.
- <u>Motion of connected blocks</u>



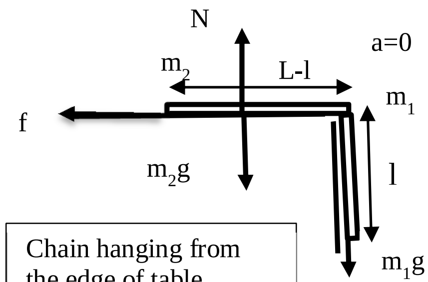
One block on table, another hanging at rest



One block on table, another hanging both sliding with acc a

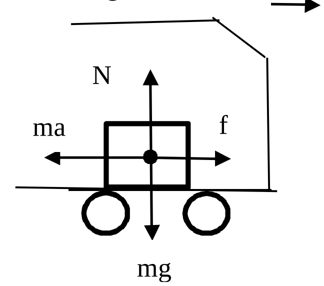
- Consider a block of mass m_1 placed on a table having coefficient of static friction μ_s. It is connected to another block of mass m_2 suspended by a pulley.
- Static frictional force is $f_s = \mu_s N = \mu_s m_1 g$.
- External force = $T = m_2 g$.
- If external force $m_2 g < f_s$, then the frictional force is $f = m_2 g$. The block does not slide.
- If external force $m_2 g = f_s$, then the frictional force is $f = f_s = m_2 g = \mu_s m_1 g$. The block tends to slide.
- $\mu_s = \frac{m_2}{m_1}$

- If external force $m_2g > f_s$, then the frictional force is $f = f_k = \mu_k m_1 g$. The block slides with acceleration. The acceleration with which the block slides is $a = \frac{m_2g - \mu_k m_1 g}{m_1}$
- Chain on table
- Consider a chain of length L and mass m is placed on a table such that a length l hangs from the edge of the table. Let m_2 be the mass of the part l hanging from the edge of the table. Let m_1 be the mass of the part (L-l) on the table. Then static frictional force is $f_s = \mu_s N = \mu_s m_2 g = \mu_s \frac{m}{L}(L-l)g$. Here $m_1 = \frac{m}{L}l$ and $m_2 = \frac{m}{L}(L-l)$



Chain hanging from the edge of table

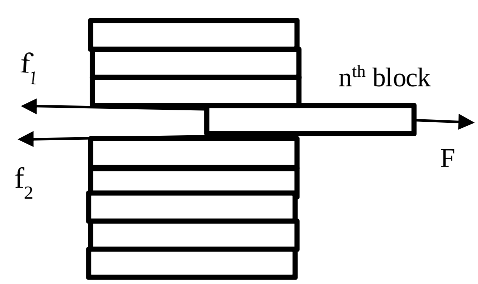
- If external force $m_1g < f_s$, then the frictional force is $f = m_1 g = \frac{m}{L}l$. The chain does not slide.
- If external force $m_1g = f_s$, then the frictional force is $f = f_s = m_1g = \mu_s m_2 g$. The chain tends to slide.
- If uniform chain of length L is placed on a table in such a way that it is just prevented from falling with a length of it hanging downwards. The coefficient of static friction between the table and the chain is $\frac{l}{L-l}$.{ $\mu_s = \frac{m_1}{m_2} = \frac{l}{L-l}$}
- If external force $m_1g > f_s$, then the frictional force is $f = f_k = \mu_k m_2 g$. The chain tends to slide.
- Block on a truck
- When a block of mass m is placed on a truck having coefficient of friction μ_s moving with an acceleration a, static frictional force is given by $f_s = \mu_s N = \mu_s mg$



Block on a truck moving with acc a

- If external force $ma < f_s$, then the frictional force is $f = ma$. The block does not slide.
- If external force $ma = f_s$, then the frictional force is $f = f_s = ma = \mu_s N = \mu_s mg$. The block tends to slide.
- If external force $ma > f_s$, then the frictional force is $f = f_k = \mu_k mg$. The block slides with acceleration.
- The block tends to slide if $f_s = ma = \mu_s mg$
- The minimum acceleration with which the truck should move so that the block does not slide is $a = \mu_s g$.

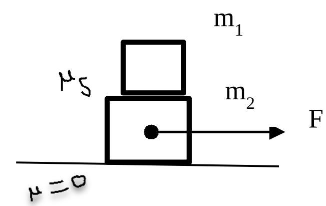
Pile of blocks



Pile of blocks

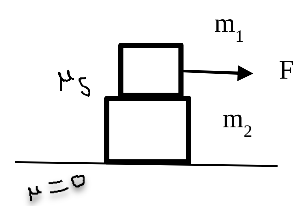
- Consider N identical blocks, each of mass m placed one above the other. The coefficient of kinetic friction between any two successive blocks is μ_k. A horizontal force F is applied on the n[th] block from the top to move it with constant speed without dislodging the others. Then frictional force between (n-1)th and nth blocks is f_1 and the frictional force between nth and (n+1)th blocks is f_2. $f_1 = \mu_k(n-1)mg$ and $f_2 = \mu_k n\, mg$. Then $F = f_1 + f_2 = \mu_k(n-1)mg + \mu_k n\, mg = \mu_k(2n-1)\, mg$

- <u>Block on block</u>
- <u>Case 1: Force on lower block</u>
- Consider a block of mass m_1 placed on a block of mass m_2. The coefficient of friction between the two blocks is μ_s and there no friction between the lower block and the surface. A horizontal force F is applied on the lower block. Then three cases arise.



Force on lower block

- Static frictional force between the upper and lower block = $f_s = \mu_s N_1 = \mu_s m_1 g$
- External force on the upper block or pseudo force $F_1 = m_1 a$ and $a = \frac{F}{(m_1+m_2)}$
- If $F_1 < \mu_s m_1 g$, both the blocks move together. $f = F_1 = m_1 a$
- If $F_1 = \mu_s m_1 g$, both the blocks move together. $f_s = \mu_s m_1 g = m_1 a$. $a = \mu_s g$
- If $F_1 > \mu_s m_1 g$, the blocks move relative to each other. $f_k = \mu_k m_1 g$. Acceleration of the upper block with respect to the ground $a_1 = \mu_k g$ and the acceleration of the lower block with respect to the ground $a_2 = \frac{F-\mu_k m_1 g}{m_2}$. The acceleration of the upper block is constant relative to the ground even when the force on the lower block is increased.
- Acceleration of the upper block relative to the lower block $F_R = m_1 a - \mu_k m_1 g$. $a_1' = a - \mu_k g$.
- <u>Case2: Force on upper block</u>
- A horizontal force F is applied on the upper block.
- Static frictional force between the upper and lower block = $f_s = \mu_s N_1 = \mu_s m_1 g$
- External force on the lower block or pseudo force on lower block = $m_2 a$ and $a = \frac{F}{(m_1+m_2)}$ where a is the acceleration of the system.



Force on upper block

- If $m_2 a < \mu_s m_1 g$, blocks move together and $f = m_2 a$
- If $m_2 a = \mu_s m_1 g$, blocks move together and $F = m_2 a = f_s = \mu_s m_1 g$. Acceleration of the blocks is $a_1 = a_2 = \frac{\mu_s m_1 g}{m_2}$
- If $m_2 a > \mu_s m_1 g$, blocks move separately and $f_k = \mu_k m_1 g$. Acceleration of the lower block with respect to ground $a_2 = \frac{\mu_k m_1 g}{m_2}$. Acceleration of the upper block with respect to ground $a_1 = \frac{F-\mu_k m_1 g}{m_1}$.